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Tutorial 4

Problem 1 Achievable data rates with IEEE 802.11a Wireless LAN

In this problem, we consider the physical layer of IEEE 802.11a (one of the WLAN standards). IEEE 802.11a carrier frequencies between 5127 MHz and 5910 MHz. Since the regulation of radio frequencies depends on the country, the available frequency ranges differ in international comparison. In Germany, for example, only the 5170 MHz to 5330 MHz range is available without restrictions. This corresponds to a bandwidth of 160 MHz, which is divided into a total of 8 channels of 20 MHz each. Each channel is in turn divided into 64 subcarriers of 312.5 kHz each (see Figure 1.1). Of these subcarriers, only 48 are used for data transmission[1].

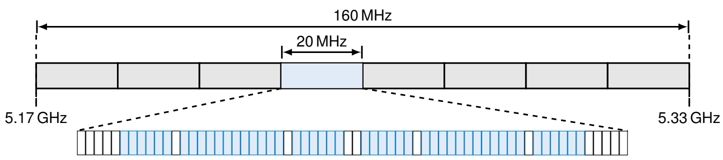

Figure 1.1: IEEE 802.11a channel distribution. Of the total of 64 subcarriers, only 48 (blue) are used for data transmission.

data rate [Mbit/s]	modulation	code rate
6	BPSK	1/2
9	BPSK	3/4
12	QPSK	1/2
18	QPSK	3/4
24	16-QAM	1/2
36	16-QAM	3/4
48	64-QAM	2/3
54	64-QAM	3/4

Figure 1.2: Data rates, modulation methods and code rates for IEEE 802.11a.

The symbol duration (temporal extension of a transmission pulse) is therefore $1/312.5\,\text{kHz} = 3.2\,\mu\text{s}$. To avoid interference from reflections, a guard interval („Guard Interval“) is inserted between symbols. The effective symbol duration is therefore $T_s = 4\,\mu\text{s}$.

The effective data rate that can be achieved depends on the modulation method used and the code rate of the channel code. These are listed in Table 1.2.

For the time being, we only consider the maximum transmission rate $r_{max} = 54\,\text{Mbit/s}$.

a)* How many bits are transmitted per symbol?

Let M be the number of different symbols, so for 64-QAM $M = 64$. Then we get per symbol

$$n = \log_2(M) = \log_2(64) = 6\,\text{bit}.$$

b) How many bits in total are transmitted per symbol duration when using 48 subcarriers?

$$n_{brutto,48} = n \cdot 48 = 288\,\text{bit}$$

c) The value calculated in Subproblem b) relates to channel words, i. e., redundancy is included. Determine the amount of user data transmitted per symbol duration.

$$n_{netto,48} = \frac{3}{4} n_{brutto,48} = 216\,\text{bit}$$

d) Using the result from Subproblem c) confirm the maximum data rate $r_{max} = 54\,\text{Mbit/s}$.

[1] The remaining subcarriers are either unused or are used for transmission of so-called pilot symbols, which are used for channel estimation. We disregard this in this problem

$$r_{\max} = \frac{n_{netto,48}}{T_s} = 216\,\text{bit} \cdot 250\,000/\text{s} = 54\,\text{Mbit/s}$$

e)* Now, using Hartley's law, determine the minimum bandwidth $B_{\min}$ necessary to achieve a data rate of 54 Mbit/s using 64 distinguishable symbols.

$$r = 2B_{\min}\log_2(M) \Rightarrow B_{\min} = \frac{r}{2\log_2(M)} = \frac{54\,\text{Mbit/s}}{2 \cdot \log_2(64)\,\text{bit}} = 4.5\,\text{MHz}$$

f) * Determine the minimum Shannon SNR in units of dB so that theoretically the maximum data rate $r_{\max} = 54\,\text{Mbit/s}$ can be achieved.
Hint: For simplicity, assume the total channel bandwidth of $B = 20\,\text{MHz}$.

$$r_{\max} \overset{!}{=} B\log_2(1 + \text{SNR})$$
$$\text{SNR} = 2^{r_{\max}/B} - 1$$
$$= 2^{(54\cdot 10^6)/(20\cdot 10^6)} - 1 = 2^{54/20} - 1 = 2^{2,7} - 1 \approx 5,50$$

Conversion to dB:

$$\text{SNR}\,\text{dB} = 10 \cdot \log(\text{SNR})\,\text{dB} \approx 7.40\,\text{dB}$$

Hinweis:

- $\log$ or $\log_{10}$ $\triangleq$ decimal logarithm
- $\log_2$ or ld $\triangleq$ binary logarithm
- ln $\triangleq$ natural logarithm

g) The signal power at the receiver is now 45 µW. The noise has a power of 15 µW. Which modulation method and which code rate will be used under these conditions?
Hint: For simplicity, assume the total channel bandwidth of $B = 20\,\text{MHz}$.

$$r = B \cdot \log_2(1 + \text{SNR})$$
$$= B \cdot \log_2\left(1 + \frac{P_S}{P_N}\right)$$
$$= 20 \cdot 10^6/\text{s} \cdot \log_2\left(1 + \frac{45}{15}\right)\,\text{bit} = 40\,\text{Mbit/s}$$

From Table 1.2 we now see that $48\,\text{Mbit/s} > r > 36\,\text{Mbit/s}$ holds. The data rate is therefore switched down to at most 36 Mbit/s. Consequently, QAM-16 and a code rate of $R = 3/4$ are used.

Problem 2 Transmission Channels

A new undersea cable has connected Japan and the USA since 2010. The cable runs from Chikura near Tokyo to Los Angeles in California (approx. 10 000 km) and consists of 8 fiber pairs (in each fiber pair, one fiber is used for one direction and the other fiber for the other direction). The transmission rate amounts to a total of 7.68 Tbit/s per direction.

As a simplifying assumption, we assume that the light only travels the path of the cable and that no signal impairments or delays occur due to signal amplifiers, connectors and the like. The relative propagation speed of light within an optical fiber (as well as in copper cables) is approximately $\nu = 2/3$ in relation to the speed of light in a vacuum $c_0 = 3 \cdot 10^8\,\mathrm{m/s}$.

a)* Determine the propagation delay from Chikura to Los Angeles within the cable.

$$t_p = \frac{d}{\nu c_0} = \frac{10^7\,\mathrm{m}}{2/3 \cdot 3 \cdot 10^8\,\mathrm{m/s}} = 50\,\mathrm{ms}$$

b)* What does the *bandwidth delay product* mean?

The bandwidth delay product specifies the „amount of data stored“on the line, i.e. how many bits are serialized by the sender before the first bit reaches the receiver.

c) Determine the bandwidth delay product.

$$B = r \cdot t_p = 7,68 \cdot 10^{12}\,\mathrm{bit/s} \cdot 50 \cdot 10^{-3}\,\mathrm{s} = 384 \cdot 10^9\,\mathrm{bit} = 384\,\mathrm{Gbit} = 48\,\mathrm{GB}$$

Laying and maintaining a submarine cable is very complex. The connection between the two cities could also be made via satellite. Take a brief look at the two connection paths in relation to the round trip time (RTT[2]).

Assume that the submarine cable is in direct airline connection between Chikura and Los Angeles. In doing so, neglect the curvature of the earth. A geostationary satellite (36 000 km altitude) is located exactly above the center of the route.

d) Determine the minimum RTT for the submarine cable. **Note:** Think about which component of the RTT makes the most significant contribution in this case.

$\mathrm{RTT} = 2 \cdot (t_s + t_p)$.With $t_s \to 0$ (very high transmission rate), the RTT is reduced to $\mathrm{RTT} = 2t_p$.

$$\mathrm{RTT} = 2t_p = 100\,\mathrm{ms}$$

[2] RTT is the time it takes for a message to travel from the sender to the recipient and back again

e) Determine the minimum RTT for a corresponding satellite connection.
Note: Consider which sections of the route can be neglected if necessary. The curvature of the earth may be neglected.

$$\mathrm{RTT}_{\mathrm{Satellite}} = 2 \cdot t_{p,sat} = 2 \cdot \frac{d_{sat}}{c_0} = 2 \cdot \frac{2 \cdot \sqrt{5000^2 + 36000^2}\,\mathrm{km}}{3 \cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}} \approx 485\,\mathrm{ms}$$