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# Computer Networking and IT Security (INHN0012)

Tutorial 3

## Problem 1 Quantization and channel noise

In this task, a temperature curve is to be digitized and the influence of noise on signals is to be investigated. For this purpose, temperatures in the range of –40 ℃ to 70 ℃ are to be considered. The measured values are to be mapped linearly, with a step size of at most 0.5 ℃.

a)* Explain the difference between sampling and quantization.

- Sampling is the discretization of a continuous signal in the time domain without rounding.
- Quantization is the discretization of a signal in signal steps, i.e. h. in the value range with rounding.

b)* What is the minimum number of bits required to digitize a single temperature value? Give reasons for your answer.

From the lecture we know the connection

$$M = \frac{b-a}{\Delta}, \tag{1.1}$$

where $M$ is the number of signal steps, $a$ and $b$ are the lower and upper limits of the quantization interval, respectively, and $\Delta$ is the step width. Substituting $M = 220$ signal levels, which in turn corresponds to $N = \lceil \log_2(M) \rceil = 8\,\text{bit}$.

c) According to the Subproblem b), which step size can be used to determine the temperature based on the number of bits used?

Since we have to use 8 bit to represent quantization levels anyway, in practice $M' = 256$ instead of $M = 220$ quantization levels. Solving (1.1) for $\Delta'$ and substituting yields $\Delta' \approx 0.43\,°\text{C}$

d) Determine the maximum quantization error with respect to the calculated step size from Subproblem c) assuming that mathematical rounding is used.

$$\Delta'/2 = 0.43\,°\text{C} \cdot \frac{1}{2} \approx 0.215\,°\text{C}$$

If you have not solved previous subproblems, assume 256 quantization levels.[1]
The baseband signal used uses exactly one symbol for each temperature level. A channel capacity of 10 kbit/s should be achieved.

e) Determine the minimum bandwidth required for a noise-free channel if the specified channel capacity is to be achieved.

Hartley's law: $C_H = 2B\log_2(M) \Rightarrow B = \frac{C_H}{2\log_2(M)} = 625\,\text{Hz}$

f) To what value would the channel capacity decrease for the same bandwidth if an SNR of 35 dB were applied?

Shannon's law: $C_S = B\log_2(1 + \text{SNR})$ where dB is 10 times the decadic logarithm of two equal quantities. So we get for the SNR:

$$\text{SNR} = 10\log(X) \Rightarrow X = 10^{(\text{SNR}/10)} \approx 3162.28$$

Substituting this into $C_S$ yields:

$$C_S = B\log_2(1 + X) \approx 7267\,\text{bit/s}$$

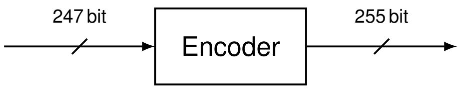
(The achievable bandwidth is always the minimum of $C_H$ and $C_S$!)



[1] In the written exam, tasks basically build on each other, i. e., intermediate results of previous subproblems are to be used. For longer tasks, we — when appropriate — sometimes give substitute values so that reentry is possible.

# Problem 2 Channel Coding

In the previous sheet we saw that the frame error probability can become a problem with poor channel quality. For the radio channel with a bit error probability $p_e = 10^{-4}$, the success probability for a frame of length 1500 B was only about 30 %. To counter the high bit error rate, a block code is now used on Layer 1:




This allows the decoder on the receiver side to correct any bit error in a channel word of length $n = 255$ bit *any*. If two or more bit errors occur, the decoder's decision is wrong and all the information of the channel word is lost.

a)* Determine the code rate.

$$R = \frac{k}{n} = \frac{247}{255} \approx 0.97$$

b)* What does the code rate mean?

The code rate expresses the ratio between the size of a user data block and the size of a user data block (channel word) secured by redundancy. The smaller $R$, the more redundancy was added. Thus, for $R = 247/255$, each channel word of length 255 bit carries a total of 8 bit of redundancy as well as 247 bit of information.

c)* Since the frame is larger than a block of 247 bit, it must be divided into several blocks. Determine the number $N$ of channel words that must be transmitted.

Each channel word of length 255 bit carries 247 bit user data. The result is therefore:

$$N = \left\lceil \frac{1500 \cdot 8}{247} \right\rceil = 49.$$

d) Padding is included in the last channel word. Determine the percentage overhead of the padding in relation to the possible user data in the channel words.

maximum possible payload = $N \cdot 247\,\text{bit} = 12\,103\,\text{bit}$.
padding = possible payload - actual payload = $12\,103\,\text{bit} - 1500\,\text{B} \cdot 8 = 103\,\text{bit}$

$$\gamma = \frac{\text{padding}}{\text{maximum possible payload}} = \frac{103\,\text{bit}}{12\,103\,\text{bit}} \approx 0.85\,\%.$$

Alternatively:

$$\gamma = \frac{\text{Padding}}{\text{actual Payload} + \text{Padding}} = \frac{103\,\text{bit}}{1500\,\text{B} \cdot 8 + 103\,\text{bit}} \approx 0.85\,\%.$$

e)* Determine the probability that a single channel word is decoded incorrectly.

The probability that a single channel word is decoded with errors corresponds to the probability that two or more errors occur within the channel word. Let $X$ be the random variable indicating the number of bit errors in a channel word of length $n$.

$$\begin{aligned} p_{e,codeword} &= \Pr[X \geq 2] = 1 - \Pr[X \leq 1] = 1 - \sum_{i=0}^{1} \binom{n}{i} \cdot p_e^i \cdot (1 - p_e)^{n-i} \\ &= 1 - \left(1 \cdot p_e^0 \cdot (1 - p_e)^{255} + 255 \cdot p_e^1 \cdot (1 - p_e)^{254}\right) \\ &\approx 1 - \left(0,9748 + 255 \cdot 10^{-4} \cdot 0,9748\right) \\ &\approx 3.18 \cdot 10^{-4} \end{aligned}$$

f) Now determine the probability that a frame will be transmitted correctly — that is, none of the channel words that make up the frame will be transmitted incorrectly.

For the frame to be transmitted correctly, all channel words must be correctly be transmitted correctly. With the results of the previous subtasks therefore:

$$\Pr[\text{„no error in frame“}] = (1 - p_{e,codeword})^N \approx 98.50\,\%$$