

Computer Networking and IT Security (INHN0012)

Tutorial 2

Problem 1 Source entropy

As we will see in the lecture, a message source can be abstractly modeled as follows:

- A message source Q emits statistically independent character from an alphabet $\mathcal{X} = \{a, b\}$.
- The source *Q* is assumend to be memoryless, i. e., the ouput in step *n* > does not depend on the output of any previous step.
- The emitted symbols *a*, *b* carry a different amount off information, depending on their probability of occurence in general, the less likely a symbol is being emitted, the higher the information content.
- Information measured in bit

We model this message source as a discrete random variable *X*. The probability that the source emits the character X = a is $p_a = \Pr[X = a] = 0.25$.

a)* Determine the probability p_b that the character X = b is emitted.

Sin	се	р _а -	- p b	= 1	1, it	foll	ow	s th	at <i>j</i>) _b =	0.	75.										

b) Determine the information content I(a) and I(b) of both symbols.

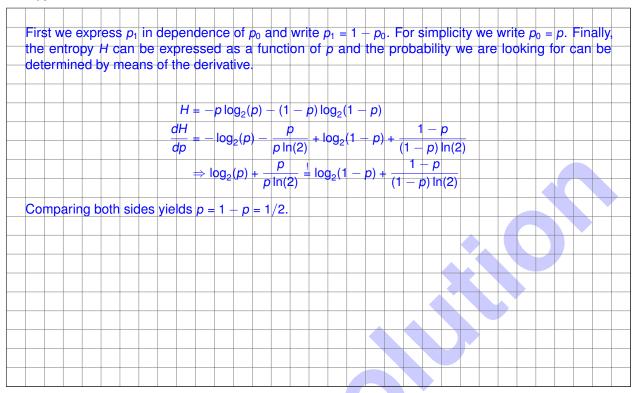
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						I	(b)	-4	log	$ _2 p_l$, ≈	0.4	2 b	t						

c) Determine the entropy *H* of the source.

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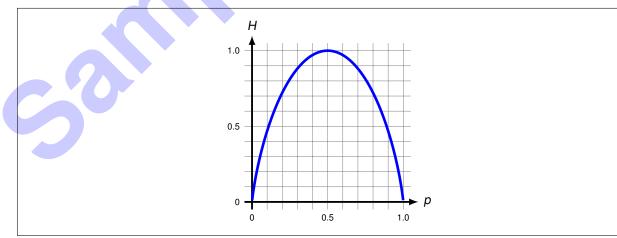
d) Determine the occurrence probabilities p_0 and p_1 of another binary message source Q' such that its entropy *H* is maximal.



e) What is the maximum entropy of a binary source?

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f) Sketch the source entropy H of a binary source in general as a function of the probability of occurrence p.



g) Obviously, the entropy H(X) < 1 is not maximal. What conclusion can be drawn from this fact for the data stream emitted by source Q with respect to redundancy?

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h) Generalise the results of the subtasks d) and e) to a source emmitting *n* different symbols.

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Problem 2 Probability of frame errors

In the lecture, the bit error probability for radio links will be given as about $p_{e,1} = 10^{-4}$ and for Ethernet over copper cable as about $p_{e,2} = 10^{-8}$. We assume that bit errors occur independently and uniformly distributed by a noise with constant power over time. Thus, the channel characteristics do not change over time. Let other disturbing influences such as interference be excluded. Let the frame length be equal 1500 B.

a)* For both types of transmission, determine the probability that a frame will be transmitted without errors.

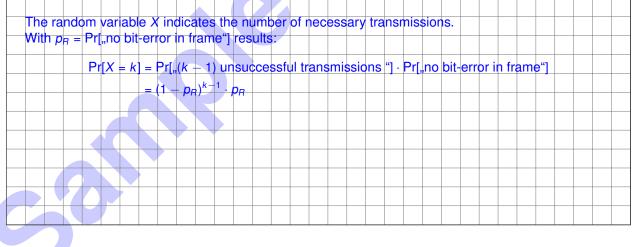
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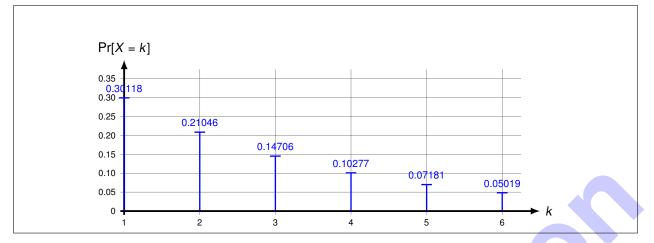
In the following, we will only consider the wireless connection. Since the frame error probability is relatively high, a protocol on the link layer provides for acknowledgements. An acknowledgement is therefore sent for correctly transmitted frames. If there is no confirmation, the sender assumes that the transmission was not successful. For the sake of simplicity, we assume that confirmations are not lost.

b)* Is there a maximum number of repetitions until a certain frame is guaranteed to have been transmitted correctly?

No. The individual transmissions are independent of each other, i.e. a frame error with the probabilities calculated in a) also occurs with each repetition. So there is always a residual risk.

c)'	Determine t	he prob	bability	that th	ne fra	me n	nust k	be tran	sfer	red	exa	ctly	' k	tim	es.		
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e) Assume that the responsible protocol on the link layer aborts the retransmission if the third transmission attempt was unsuccessful. What is the probability that a frame cannot be transmitted?

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